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Corrections to the Asymptotic Holtsmark Formula for Hydrogen Lines Broadened by Electrons and Ions in a Plasma*

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Abstract

Expressions are given for the electron contribution to the Stark broadening at large distances from the line center, referred to the quasi-static broadening by ions. Depending on distance from the line center and velocity, the electron contribution is calculated with the impact approximation using Lewis or Debye cutoffs, or with the quasi-static approximation. In addition to monopole-dipole interactions causing perturbations within the group of levels having the principal quantum number of the upper state of the line, some allowance is made for monopole-quadrupole interactions, for collision-induced transitions involving changes of principal quantum numbers and for perturbations of the lower state of the line. Using improved estimates for the relevant atomic matrix elements, the correction factor to the asymptotic Holtsmark result for ion broadening is calculated to an estimated accuracy of 10% for almost all lines of the early hydrogen line series, and estimates are given for higher order terms in the asymptotic expansion and Debye shielding effects. The electron broadening, while at most about equal to quasi-static ion broadening, turns out to be significant under all practical conditions, even when these lie well in the validity regime of the impact approximation.

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I. INTRODUCTION

While the Stark broadening of hydrogen lines by ions is almost always adequately described by the quasi-static (Holtsmark) approximation (high principal quantum number $n\alpha$ lines are a notable exception [Griem 1966]), if necessary corrected for Debye shielding and ion-ion correlations, the situation for electron broadening is much more complicated. The prevailing view had been that their contribution was only important when their effects could be described by the quasi-static approximation as well. However, detailed calculations [Griem et al. 1959, 1962] based on suitable versions [Baranger 1958, Kolb and Griem 1958] of the impact approximation showed that this was not the case, at least not for early members of the Lyman and Balmer series. For early Balmer lines, these calculations tended to agree with experiments within the estimated theoretical errors of 10 to 20% [Griem 1964, Wiese 1965], verifying that electron impact broadening may be just as important as quasi-static ion broadening.

The above statements mainly refer to experiments at rather high electron densities ($>10^{16}cm^{-3}$) and measurements of the central portions of the line shapes. At lower densities ($\sim10^{13}cm^{-3}$) and further out on the line wings, measurements of high members of the Balmer series [Bergstedt et al. 1961, Ferguson and Schlüter 1963, Vidal 1964] and Paschen series [Vidal 1965] all indicated that the various theoretical estimates of the electron impact broadening for such lines [Griem 1960, 1962] were too high. However, the question was left open whether or not this was due to some basic limitation in the impact broadening theory or to errors made in extrapolating from the detailed calculations for the low series members [Griem et al. 1959, 1962]. It is the purpose of the present communication to point out that the latter is the case and that estimates for the wing broadening of high members of the hydrogen line series based on the impact and quasi-static approximations, which ever is appropriate, can be made with about the same reliability as, e.g., for Lyman-α [Griem 1965], i.e., typically to within ±10%. The essential result is that electron broadening never significantly exceeds that predicted by the Holtsmark theory, but that it comes very close to this limit even under conditions for which the quasi-static approximation is not at all valid.

II. THEORY

Assuming only monopole-dipole interactions between perturbing particles and hydrogen atoms and neglecting both lower state broadening and contributions from states of different principal quantum number than the upper state of the line, quasi-static and impact approximation yield the asymptotic line shapes [see equations (7) and (19) of Griem 1962]

$$I_s \sim 2\pi N_s \sum_{\alpha\beta\sigma}{}' \langle\alpha|\mu_\sigma|\beta\rangle\langle\beta|\mu_\sigma|\alpha\rangle \left(\frac{e^2}{\hbar}\langle\beta|r_3|\beta\rangle\right)^{3/2} |\Delta\omega|^{-5/2}, \tag{1}$$

$$I_i \sim \frac{4}{3v}\left(\frac{e^2}{\hbar}\right)^2 N_i \sum_{\alpha\beta\beta'\beta''\sigma\nu} \langle\alpha|\mu_\sigma|\beta\rangle\langle\beta'|\mu_\sigma|\alpha\rangle \tag{2}$$

$$\times \left(E_{\beta\beta'} + \ln\frac{\rho_{max}}{\rho_{min}}\right)\langle\beta|r_\nu|\beta''\rangle\langle\beta''|r_\nu|\beta'\rangle|\Delta\omega|^{-2}.$$

Here N_s and N_i are the number densities of quasi-static and impact broadening perturbers, v is the velocity of the latter, μ_σ and r_ν are dipole and coordinate vector operator components of the atomic electron, and $\Delta\omega$ is the angular frequency separation from the unperturbed line. The sums are over the various sublevels α and β, β', β'' of the lower and upper states of the line (in case of the quasi-static formula only over half of all components, namely those which are shifted to one side of the unperturbed line) and over the various components of μ_σ and r_ν. Finally, e is the charge of the electron and $\hbar$ Planck's constant divided by 2π.

The quantities $E_{\beta\beta'}$ in equation (2) were estimated to be or order $\delta_{\beta\beta'}$ and actually determined from the requirement that there should be a smooth transition from quasi-static to impact approximations [Griem 1962]. Alternatively, $E_{\beta\beta'}$ may be obtained directly by calculating the so-called strong-collision term, i.e., by carrying the iterative solution of the time-dependent Schrödinger equation, whose second-order solution gives rise to the logarithmic term, to higher orders. For Lyman-α this procedure results in $E_{\beta\beta'} \approx 0.29\ \delta_{\beta\beta'}$ [Griem 1965], while estimates of the strong collision term based on the second order solution give $E_{\beta\beta'} \approx 0.50\ \delta_{\beta\beta'}$. An intermediate value will be chosen here in order to account, at least roughly, also for monopole-quadrupole interactions which give rise to an additional term of order kT/E_H [Griem 1965], E_H being the ionization energy of hydrogen. The constant preceding the logarithm should then typically be within ± 0.1 of its true value.

The minimum impact parameter under the logarithm corresponds to the breakdown of second-order perturbation theory. It is of order [Griem et al. 1959]

$$\rho_{min} \approx n^2 \frac{\hbar}{mv}, \tag{3}$$

where n is the principal quantum number of the upper state. The maximum impact parameter is introduced to account for the finite duration of the collisions [Lewis 1961] or for Debye-shielding [Griem et al. 1959] and is therefore chosen as [Griem 1962, 1965]

$$\rho_{max} \approx \mathrm{Min}\ \frac{v}{|\Delta\omega|}, \left(\frac{kT}{4\pi Ne^2}\right)^{1/2}, \tag{4}$$

i.e. as the minimum of the length an electron can travel in times contributing to the Fourier integral which represents the line shape, and the Debye length appropriate for shielding by other electrons.

In the calculation of the velocity average of the impact profile, the Lewis cutoff is thus to be used for small velocities and the Debye cutoff for large velocities. Also, the integral over velocities should not begin before ρ_{max} becomes larger than ρ_{min}. Assuming a Maxwell-Boltzmann distribution for the electron velocities and introducing a new variable $y = \frac{1}{2} mv^2/kT$ the integration (average) of equation (2) is readily performed [Griem 1965] yielding

$$I_1 \sim \frac{4}{3} \left(\frac{e^2}{\hbar}\right)^2 \left(\frac{2m}{\pi kT}\right)^{1/2} N_1 \sum \langle\alpha|\mu_\zeta|\beta\rangle\langle\beta'|\mu_\zeta|\alpha\rangle \tag{5}$$

$$\times \left(\frac{2}{5}\,\delta_{\beta\beta'} e^{-y_1} + \int_{y_1}^{\infty} \frac{e^{-y}}{y}\,dy - \frac{1}{2}\int_{y_2}^{\infty} \frac{e^{-y}}{y}\,dy\right) \langle\beta|r_\nu|\beta''\rangle\langle\beta''|r_\nu|\beta'\rangle |\Delta\omega|^{-2} .$$

The quantity y_1 corresponds to the velocity where minimum impact parameter and Lewis cutoff are equal to each other, and y_2 to the velocity where Lewis and Debye cutoff are interchanged, i.e.,

$$y_1 = \frac{n^2 \hbar |\Delta\omega|}{2kT} , \tag{6}$$

$$y_2 = \frac{m|\Delta\omega|^2}{8\pi N e^2} . \tag{7}$$

Electrons for which the minimum impact parameter is larger than the Lewis cutoff, i.e., for which $y < y_1$, actually fulfill [Griem 1962] the usual validity criterion for the quasi-static approximation, and should therefore be counted as contributing toward the density of quasi-static perturbers. This consideration leads to

$$N_s = N\left(1 + \frac{2}{\sqrt{\pi}} \int_0^{y_1} y^{1/2} e^{-y} dy\right) \tag{8}$$

$$= N\left[1 + \frac{4}{3\sqrt{\pi}} y_1^{3/2}\left(1 + 3 \sum_{n=1}^{\infty} \frac{(-1)^n y_1^n}{(2n+3)n!}\right)\right]$$

[Griem 1965], always assuming that electron and ion densities are the same (N), which will be the case unless multiple ionization is important. Other implicit assumptions are that y_1 is larger than y_2, which seems to be true in all practical situations, and again that the electron velocity distribution is Maxwellian.

It now only remains to estimate the various atomic matrix elements entering equations (1), (2), and (5). The matrix of $\sum_{\beta'' \nu} r_\nu |\beta''\rangle\langle\beta''| r_\nu$ is diagonal in spherical coordinates, and its elements are [Griem 1966] in terms of the Bohr radius a_o,

$$\sum_{\ell'' m'' \nu} \langle n\ell m|r_\nu|n\ell'' m''\rangle\langle n\ell'' m''|r_\nu|n\ell m\rangle = \frac{9}{4} a_o^2 n^2 (n^2-\ell^2-\ell-1). \tag{9a}$$

Because of the selection rules for the dipole (μ) matrix elements, the largest value of the orbital quantum number ℓ is equal to the principal quantum number n' of the lower state. For high series members equation (9a) may thus be replaced by

$$\sum_{\ell'' m'' \nu} \langle n\ell m|r_\nu|n\ell'' m''\rangle\langle n\ell'' m''|r_\nu|n\ell m\rangle \approx \frac{9}{4} a_0^2 n^4 . \tag{9b}$$

The ratios of the mean values of equation (9a), using $|\langle n\ell m|\mu|n'\ell' m'\rangle|^2$ as weight factors, and of equation (9b) have been calculated for the Lyman, Balmer and Paschen series with the aid of exact radial matrix elements [Green et al. 1957] and are listed in Table 1. Inspection of the tabulated ratios indicates that the use of equation (9b) causes errors of less than 20% in the impact broadening of the upper levels of all but the first two Lyman, three Balmer and four Paschen lines. Also listed (in parentheses) are the ratios of average matrix elements referred to matrix elements extrapolated [see equation (21) of Griem 1960] from exact values for the first two Lyman and first four Balmer lines [Griem et al. 1959]. While these extrapolated matrix elements indeed give ratios close to unity for most of the early series members, they are too large by a factor of about $n/6.75$ for high series members, i.e., the electron impact broadening had been over-estimated considerably for these lines.

The matrix elements entering the quasi-static formula, on the other hand, are well estimated by

$$\sum_{\alpha\beta}{}' \langle\alpha|\mu\ |\beta\rangle\langle\beta|\mu\ |\alpha\rangle\left(\frac{e^2}{\hbar}\ \langle\beta|r_3|\beta\rangle\right)^{3/2} \approx \frac{3}{8}\left(\frac{\hbar}{m}\right)^{3/2} n^3 \qquad (10)$$

[see equation (19) of Griem 1960], with errors of only a few percent for high series members. The final result for the total asymptotic wing intensity is best expressed relative to the asymptotic Holtsmark result for the ion contribution. The latter is given by equation (1) with $N_s = N$, and the total wing intensity, i.e., the sum of ion and electron contributions, follows by adding equations (1) and (5), determining the value of N_s from equation (8). In this manner and using equations (6), (9b), and (10) the correction factor to the asymptotic Holtsmark result becomes

$$f \approx 1 + \frac{2}{\sqrt{\pi}} \int_0^{y_1} y^{1/2} e^{-y}\, dy + \left(\frac{4}{\pi}\right)^{3/2} y_1^{1/2} \left(\frac{2}{5} e^{-y_1} + \int_{y_1}^{\infty} \frac{e^{-y}}{y}\, dy - \frac{1}{2} \int_{y_2}^{\infty} \frac{e^{-y}}{y}\, dy\right) \left(1 - \frac{n'^2}{n^2}\right)^{1/2} \qquad (11)$$

with y_1 and y_2 defined through equations (6) and (7). Also, to at least approximately account for the perturbations of the lower state (principal quantum number n'), the electron impact contribution was multiplied with the factor $(1 - n'^2/n^2)^{1/2}$. This is suggested by the much improved agreement with exact calculations for low series members when the factor n^3 in equation (10) is replaced by $(n^2 - n'^2)^{3/2}$ [Griem 1960] and by the observation that the electron impact broadening is reduced by essentially a factor $(1 - n'^2/n^2)^2$ [Griem 1966] by lower state perturbations.

III. DISCUSSION

The second term in equation (11) represents the relative correction to the quasi-static ion broadening for quasi-static electron broadening, the third term that for electron impact broadening. For $y_1 << 1$, electron impact broadening dominates in the correction, for $y_1 >> 1$ quasi-static electron broadening. (Values of y_1 for ions are larger than those for electrons by the ion to electron mass ratio, if electron and ion temperatures are the same, and it is therefore usually safe to use the quasi-static approximation for ion broadening as long as the electron y_1 is larger than, say, 10^{-3}, which is generally the case outside of the Doppler cores.) The correction factor as given by equation (11) but without the factor $(1 - n'^2/n^2)^{1/2}$ is shown in Figure 1 as function of y_1 and for various values of the parameter y_2. The most important feature of these curves is that f remains considerably above the value f = 1 even for small y_1 for which the usual validity criteria for the quasi-static approximation are violated by an order of magnitude or more, as long as y_2 is significantly larger than y_1. That this is usually the case can be seen by estimating the ratio of y_2 and y_1 for points outside of the Doppler core, i.e., for $|\Delta\omega| > (kT/Mc^2)^{1/2}\omega$, where M is the hydrogen atom mass and $\omega = 2\pi c/\lambda$ the angular frequency of the line. This ratio is then from equations (6) and (7) and combining the various constants into the hydrogen ionization energy and Bohr radius,

$$\frac{y_2}{y_1} > \frac{1}{2}\left(\frac{kT}{2E_H}\right)^{3/2}\left(n^2\lambda a_o^2 N\right)^{-1} , \tag{12}$$

which indeed exceeds unity by a large factor for all experimental conditions yet reported.

Even though equation (11) was derived essentially for high series members, it should be reasonably accurate for many of the early lines as well. This contention is supported by the good agreement (to within a few percent) with more detailed calculations for Lyman-α [Griem 1965], which suggests that errors from the use of approximate matrix elements should be well below 10% for all Lyman, Balmer and Paschen lines except, perhaps, for H_α and the first two Paschen lines. (This surprisingly good accuracy comes from a near-cancellation of errors in the matrix elements for impact and quasi-static broadening.) Other theoretical errors in equation (11) are connected with uncertainties in the definitions of y_1 and y_2 and the estimated uncertainty of ± 0.1 in the factor $\frac{2}{5}$ of the strong collision term. However, even if one allows for a factor of 2 uncertainty in y_1 and y_2, the combined error from these sources stays practically always below 10%. Additional errors are, in principle, incurred by neglecting perturbations involving intermediate states of different principal quantum numbers. These must be included as soon as the angular frequency splitting between neighboring states, $\omega_{n,\ n+1} \approx 2E_H/\hbar n^3$, is less than $v/\rho_{min} \approx mv^2/\hbar n^2$, using equation (3). Estimates analogous to those performed for high principal quantum number $n\alpha$ lines [Griem 1966] indicate* that then a term $\frac{1}{3}\int_{y_3}^{\infty} e^{-y}dy/y$ should be added to the factor containing the exponential integrals in equation (11), with

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* Average matrix elements of $\sum_{n'',\ell'',m'',\nu} r_\nu |n''\ell''m''\rangle\langle n''\ell''m''| r_\nu$ summed over all $n'' \neq n$ are for high series members 1/9 of the average matrix elements with $n'' = n$. However, the dominant contribution to the wing intensity from collision-induced transitions to $n'' \neq n$ corresponds to a combination of dipole (μ) matrix elements for the transitions $n' \rightarrow n$ and $n' \rightarrow n+1$, which are for high series members almost equal to each other, and r_ν matrix elements for the transitions $n \rightarrow n$ and $n \rightarrow n+1$. The relevant combination of matrix elements is therefore about $1/\sqrt{9}$ of that for $n'' = n$.

$y_3 \approx E_H/(nkT)$, to account for collision-induced transitions to different principal quantum number levels.

After this correction, theoretical uncertainties in the ratio of total wing intensities due to Stark broadening to the intensity predicted by the asymptotic Holtsmark formula for ion broadening are accordingly less or about 10% for almost all lines of the early hydrogen line series. In practice, additional errors may be incurred by applying asymptotic formulas too near to the line center, where higher order terms in the asymptotic expansion for the line shape are important and where it is often necessary to correct for Debye shielding, etc., of the quasi-static perturbations, or too far out on the line wings where overlap with the next line in the series is considerable. The second difficulty is usually immediately recognizable, while the former may be concealed by a near-cancellation of the two corrections, both of which affect to first approximation only the quasi-static broadening. For Lyman-α [Griem 1965] these (relative) corrections to the quasi-static broadening were estimated as

$$\frac{\Delta I_s}{I_s} \approx \frac{32}{5}\left(\frac{2}{\pi}\right)^{1/2}\left|\frac{CF_o}{\Delta\omega}\right|^{3/2} + \frac{5}{2}\left|\frac{w}{\Delta\omega}\right| - \frac{5}{4}\left|\frac{Ce}{r_s^2\Delta\omega}\right| . \qquad (13a)$$

Here C is the linear Stark effect constant relating the frequency shift $\Delta\omega_s$ to the field strength F through $\Delta\omega_s = CF$, $F_o \approx (4\pi/3)^{2/3} eN_s^{2/3}$ is the Holtsmark normal field strength, w the half width of the impact profile which will here be related to the asymptotic impact profile through $I_i \sim w/(\pi\Delta\omega^2)$, and r_s is the effective shielding length which should lie somewhere between the Debye radii accounting for shielding by electrons only or by ions and electrons. Choosing therefore $r_s^{-2} \approx 6\pi Ne^2/kT$,

$$w \approx \frac{3n^4}{\pi} \left(\frac{\hbar}{m}\right)^2 \left(\frac{2m}{\pi kT}\right)^{1/2} N\left(\frac{2}{5} e^{-y_1} + \int_{y_1}^{\infty} \frac{e^{-y}}{y} dy - \frac{1}{2} \int_{y_2}^{\infty} \frac{e^{-y}}{y} dy\right)$$

and in analogy to equation (10) $C \approx \left(\frac{3}{8}\right)^{2/3} \frac{\hbar n^2}{me}$ and using equation (6) the relative correction is estimated to be

$$\frac{\Delta I_s}{I_s} \approx 2\pi \frac{E_H n^4}{kTy_1} Na_o^3 \left[1.3 \left(\frac{E_H n^4}{kTy_1}\right)^{1/2} \left(1 + \frac{2}{\sqrt{\pi}} \int_0^{y_1} y^{1/2} e^{-y} dy\right)\right.$$

$$\left. + 0.43n^2 \left(\frac{E_H}{kT}\right)^{1/2} \left(\frac{2}{5} e^{-y_1} + \int_{y_1}^{\infty} \frac{e^{-y}}{y} dy - \frac{1}{2} \int_{y_2}^{\infty} \frac{e^{-y}}{y} dy\right) - 3.9 \frac{E_H}{kT}\right]. \tag{13b}$$

For asymptotic formulae to be applicable $\Delta I_s/I_s$ must of course remain small and y_1 should accordingly always fulfill

$$y_1 \gg n^4 \frac{E_H}{kT} N^{2/3} a_o^2 ,$$

assuming the first term in the square bracket dominates.

To summarize, the ratio of actual to asymptotic Holtsmark wing intensities of hydrogen lines is predicted to be

$$f = \left(1 + \frac{2}{\sqrt{\pi}}\int_0^{y_1} y^{1/2} e^{-y} dy\right)\left(1 + \frac{\Delta I_s}{I_s}\right) + \left(\frac{4}{\pi}\right)^{3/2} y_1^{\ 1/2}\left(\frac{2}{5} e^{-y_1} + \int_{y_1}^{\infty} \frac{e^{-y}}{y} dy - \frac{1}{2}\int_{y_2}^{\infty} \frac{e^{-y}}{y} dy + \frac{1}{3}\int_{y_3}^{\infty} \frac{e^{-y}}{y} dy\right)\left(1 - \frac{n'^2}{n^2}\right)^{1/2} \qquad (14)$$

to within an estimated theoretical error of less or about 10%, provided both $\frac{\Delta I_s}{I_s}$ from equation (13b) and $(\frac{n'}{n})^2$ in the correction for lower state perturbations are not much larger than ~ 0.1. (Uncertainties in the correction for collision-induced transitions to different principal quantum number states should then never be important.) The quantities y_1 and y_2, now in terms of relative wavelength separations $\Delta\lambda/\lambda$ from the line center, are

$$y_1 = \frac{E_H}{2kT}\left(\frac{n^2}{n'^2} - 1\right)\left|\frac{\Delta\lambda}{\lambda}\right| , \qquad (15)$$

$$y_2 = \left(32\pi a_o^3\ N\right)^{-1}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)^2\left|\frac{\Delta\lambda}{\lambda}\right|^{\ 2} \qquad (16)$$

and y_3 [which like y_2 ought to be larger than y_1 for equation (14) to be applicable] is

$$y_3 = \frac{E_H}{nkT} \qquad (17)$$

The other symbols stand, as before, for the ionization energy of hydrogen (E_H), the electron temperature in energy units (kT), the electron density (N, assumed to be equal to the ion density), the Bohr radius (a_o) and the principal quantum numbers of upper (n) and lower (n') levels of the line, whose unperturbed wavelength is λ.

That the correction for electron broadening is indeed important for many astrophysical situations is perhaps best demonstrated by an example. Consider, e.g., the H_γ line at $N \approx 10^{14} cm^{-3}$, $T \approx 6000^o k$. About 2Å from the line center the various characteristic parameters are then $y_1 \approx 3 \times 10^{-2}$, $y_2 \approx y_3 \approx 5$, and the correction factor is from equation (14) $f \approx 1.7$. This is in fair agreement with the first calculations of the electron impact broadening correction [Griem et al. 1959], which resulted in a factor of ~ 2.05 for these conditions, and with a modified calculation [Griem 1962] accounting for the Lewis cutoff [Lewis 1961], which yielded a factor ~ 2.0. Only at larger wavelength separations from the line center the first calculations were a considerable overestimate, until the Lewis modification was included. For other early Balmer lines the situation is similar. However, as mentioned before, previous estimates for high series members [Griem 1960] were based on an incorrect extrapolation of atomic matrix elements from small to large principal quantum numbers and should thus definitely be replaced by the present calculations. Finally, there will be many cases for which equation (14) results in $f \approx 2$ over the whole wavelength range of interest (e.g., for $\Delta\lambda \gtrsim 4\text{Å}$ in the above example), justifying a posteriory the use of the asymptotic Holtsmark formula for both ions and electrons. Such asymptotic Holtsmark profiles can be obtained from tabulations of complete Holtsmark profiles [Underhill and Waddell 1959] or for lines not tabulated from equation (20) of Griem [1960], which is valid for all but the first few lines of the Lyman, Balmer, Paschen and Brackett series.

Table I

Ratio of averaged matrix elements for electron impact broadening from equation (9a) and their asymptotic values from equation (9b) for the Lyman (n' = 1), Balmer (n' = 2), and Paschen (n' = 3) series. (Values in parentheses are ratios referred to a previous estimate of the matrix elements [Griem 1960].)

n	n' = 1	n' = 2	n' = 3
2	0.250(0.819)	-	-
3	0.667(1.51)	0.368(0.731)	-
4	0.812(1.37)	0.661(1.08)	0.323(0.370)
5	0.880(1.19)	0.788(1.05)	0.593(0.743)
6	0.917(1.13)	0.854(0.956)	0.725(0.792)
7	0.939(0.906)	0.893(0.860)	0.802(0.762)
8	0.953(0.805)	0.919(0.775)	0.850(0.713)
9	0.963(0.722)	0.936(0.703)	0.882(0.660)
10	0.970(0.661)	0.948(0.639)	0.905(0.611)
11	0.975(0.599)	0.957(0.588)	0.922(0.566)
12	0.979(0.550)	0.964(0.542)	0.935(0.526)
13	0.982(0.510)	0.969(0.502)	0.944(0.490)
14	0.985(0.475)	0.974(0.470)	0.952(0.459)
15	0.987(0.444)	0.977(0.440)	0.958(0.431)
16	0.988(0.416)	0.980(0.413)	0.963(0.406)
17	0.990(0.393)	0.982(0.390)	0.968(0.384)
18	0.991(0.372)	0.985(0.370)	0.971(0.364)
19	0.992(0.352)	0.986(0.350)	0.974(0.346)
20	0.993(0.335)	0.987(0.333)	0.977(0.330)

REFERENCES

1. Baranger, M. 1958, Phys. Rev. 111, 494; 112, 855.

2. Bergstedt, K., Ferguson, E., Schlüter, H., and Wulff, H. 1961, "Proceedings of the Fifth Int. Conf. on Ionization Phenomena in Gases", I, 437, North Holland Publishing Co., Amsterdam.

3. Ferguson, E., and Schlüter, H. 1963, Ann. Phys. 22, 351.

4. Green, L. C., Rush, P. R., and Chandler, C. D. 1957, Ap. J. Suppl. 26 III, 37.

5. Griem, H. R., Kolb, A. C., and Shen, K.Y. 1959, Phys. Rev. 116, 4.

6. Griem, H. R., Kolb, A. C., and Shen, K.Y. 1962, Ap. J., 135, 272.

7. Griem, H. R., 1960, Ap. J. 132, 883.

8. Griem, H. R. 1962, Ap. J. 136, 422.

9. Griem, H. R., 1964, Ch.s 4 and 15 in Plasma Spectroscopy, McGraw-Hill Book Co., New York.

10. Griem, H. R. 1965, Phys. Rev. 140, A1140 (see also Errata, Phys. Rev. 144, 366).

11. Griem, H. R. 1966, Ap. J. (submitted).

12. Kolb, A. C., and Griem, H. R. 1958, Phys. Rev. 111, 514.

13. Lewis, M. 1961, Phys. Rev. 121, 501.

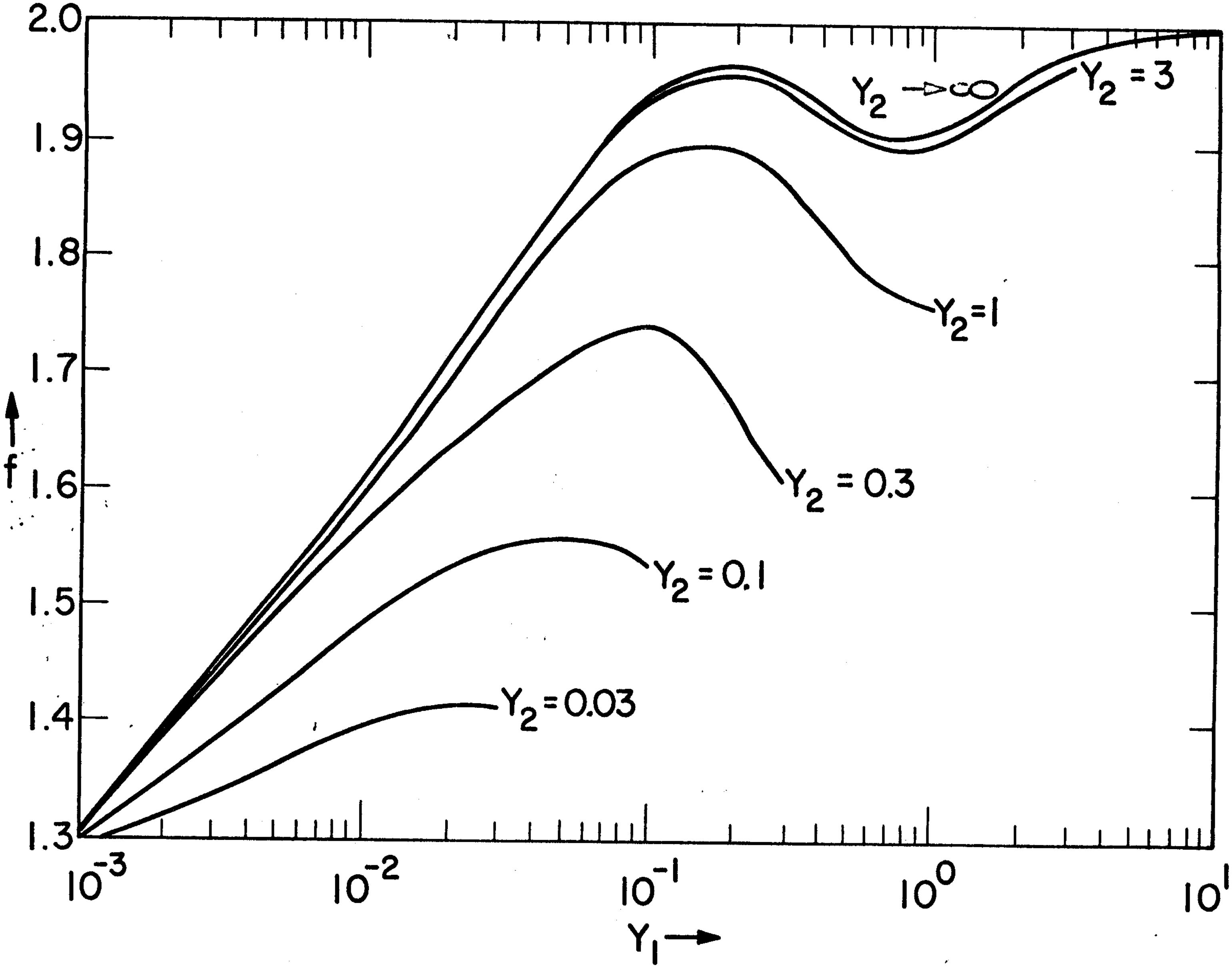
14. Underhill, A., and Waddell, J. 1959, N. B. S. Circ, No. 603.

15. Vidal, C. R. 1964, Z. Naturforschung 19a, 947; 1965, "Proceedings of the Seventh Int. Conf. on Phenomena In Ionized Gases," Belgrade (in press).

16. Wiese, W. L. 1965, Ch. 6 in R. H. Huddlestone and S. L. Leonard, Plasma Diagnostic Techniques, Academic Press, New York.

Figure 1 -

Electron broadening correction factor to the asymptotic Holtsmark formula for ion broadening, omitting corrections for lower state broadening, collision-induced transitions to different principal quantum number levels and higher order terms in the asymptotic expansion.



2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
f
$Y_2 \rightarrow \infty$
$Y_2 = 3$
$Y_2 = 1$
$Y_2 = 0.3$
$Y_2 = 0.1$
$Y_2 = 0.03$
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
Y_1